A REFINEMENT OF THE CROSSOVER SELECTION IN THE DIFFERENTIAL EVOLUTION METHOD

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Abstract. *A refinement of the crossover selection in the differential evolution method (DEM) for global optimization and some benchmarks in order to test and compare the results have been proposed. In the tested cases the introduced refinements lead to better results (in terms of mean and standard deviation) relative to the original DEM.*

Key words: Global optimization, Differential evolution method, Benchmark.

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1. Introduction

The differential evolution (DE) method [12] is widely applied for solving global optimization problems, with implementations in: Pyton library SciPy (module scipy.optimize), ALGLIB library (for C++, C#, Java, Pyton, ...), CAS `Mathematica` (as an option in NMinimize), CAS `Maple` (as one of the two methods in the GlobalOptimization Tool under the name "diffevol"), etc. In particular we use it in solving various nonlinear least squares problems with constraints (e.g. [2, 11, 13]), i.e. (in general) determining a global minimum of a function $f(\boldsymbol{x})$, where $\boldsymbol{x} = (x_1, \ldots, x_d) \in \overline{\mathcal{D}}$ and $\overline{\mathcal{D}}$ is a closed hyperrectangle in $\mathbb{R}^d$:

$$l_j \leq x_j \leq u_j, \qquad j = 1, \ldots, d.$$

While the modifications of the DE method (see [5, 6, 7]) usually concern the way mutation vector is constructed, in Section 3 we propose a refinement for the choice of the trial vector. This modification is then tested and compared with the unmodified DE method over some standard functions used in global optimization studies.

2. Outline of DE method

The differential evolution is a stochastic evolutionary algorithm for determining a global minimum of a function $f : \overline{\mathcal{D}} \to \mathbb{R}$,, where $\overline{\mathcal{D}} = \{\boldsymbol{x} =$

$(x_1, \ldots, x_d) \in \mathbb{R}^d \,|\, l_i \leq x_i \leq u_i,\ i = 1, \ldots, d\}$. It has **three parameters**:

- NP – population size (number of agents), $NP \geq 4$;
- CR – crossover probability, $CR \in [0, 1]$;
- F – differential weight, $F \in [0, 2]$.

The default values of these parameters in scipy.optimize are: $NP = 10d$, $CR = 0.9$, $F = 0.8$.

The algorithm is initialized with a generation of an **initial population** (agents) $\boldsymbol{x}_{1,0}, \ldots, \boldsymbol{x}_{NP,0}$ – a **low discrepancy sequence** of NP points in $\overline{\mathcal{D}}$. (e.g. Halton, Sobol, Latin hypercubes).

Then an **iterative** procedure starts: While [preset criterion] **is not met** do:

Iteration k ($k = 0, 1, \ldots$): For every agent, $\boldsymbol{x}_{j,k}$, in the k-th population the following is performed:

- Three different agents $\boldsymbol{a}_{j,k}$, $\boldsymbol{b}_{j,k}$ and $\boldsymbol{c}_{j,k}$ are randomly chosen:

  $$\boldsymbol{a}_{j,k}, \boldsymbol{b}_{j,k}, \boldsymbol{c}_{j,k} := \mathrm{rnd}\{\boldsymbol{x}_{1,k}, \ldots, \boldsymbol{x}_{NP,k}\} \setminus \{\boldsymbol{x}_{j,k}\} \text{ and}$$
  $$\boldsymbol{a}_{j,k} \neq \boldsymbol{b}_{j,k} \neq \boldsymbol{c}_{j,k} \neq \boldsymbol{a}_{j,k}.$$

  A **mutation vector** is constructed:

  $$\boldsymbol{\nu}_{j,k} := \boldsymbol{a}_{j,k} + F \cdot (\boldsymbol{b}_{j,k} - \boldsymbol{c}_{j,k}), \quad j = 1, \ldots, NP.$$

- A **crossover vector** (**trial vector**), $\boldsymbol{u}_{j,k}$, is defined as follows. For each component of the vector $\boldsymbol{x}_{j,k}$ a random number, $R_{i,j,k} \in [0, 1]$, is generated. If $R_{i,j,k} \leq CR$, then $u_{i,j,k} := \nu_{i,j,k}$. Otherwise: $u_{i,j,k} := x_{i,j,k}$. (In order to be ensured a difference between $\boldsymbol{x}_{j,k}$ and $\boldsymbol{u}_{j,k}$, additionally a random index is chosen for forced change.) Example: If $R_{i_0,j,k} \leq CR$ and $R_{i,j,k} > CR$, $\forall i \in \{1, \ldots, d\} \setminus \{i_0\}$. Then:

  $$\begin{aligned} \boldsymbol{x}_{j,k} &= (x_{1,j,k}, x_{2,j,k}, \ldots, x_{i_0,j,k}, \ldots, x_{d,j,k}) - \textbf{agent}, \\ \boldsymbol{\nu}_{j,k} &= (\nu_{1,j,k}, \nu_{2,j,k}, \ldots, \nu_{i_0,j,k}, \ldots, \nu_{d,j,k}) - \textbf{mutant}, \\ \boldsymbol{u}_{j,k} &= (\boldsymbol{\nu}_{1,j,k}, x_{2,j,k}, \ldots, \nu_{i_0,j,k}, \ldots, x_{d,j,k}) - \textbf{crossover}. \end{aligned}$$

- Define the agents for the $(k+1)$-th population:

  $$\boldsymbol{x}_{j,k+1} := \begin{cases} \boldsymbol{u}_{j,k}, & \text{if } \boldsymbol{u}_{j,k} \in \overline{\mathcal{D}} \text{ and } f(\boldsymbol{u}_{j,k}) \leq f(\boldsymbol{x}_{j,k}), \\ \boldsymbol{x}_{j,k}, & \text{otherwise.} \end{cases} \qquad \textbf{selection}$$

From the last generated population the best (in the sense of satisfying a preset minimality criterion) agent is selected.

Remark 2.1. *The agent $\boldsymbol{x}_{j,k}$ and the mutation vector $\boldsymbol{\nu}_{j,k}$ determine hyperrectangle (as its opposite vertices). We shall call it a "***trials cuboid***", since its vertices (excluding the agent) are the possible trial vectors corresponding to the agent $\boldsymbol{x}_{j,k}$.*

3. Refinement of the crossover selections and some benchmark

For every agent $\boldsymbol{x}_{j,k}$ the above described DE algorithm defines a corresponding crossover vector $\boldsymbol{u}_{j,k}$.

If f is a C^2 objective function, we propose the following two refinements for the crossover vector:

$$\begin{aligned}\boldsymbol{u}^u_{j,k} &:= \text{L-BFGS-B}(f(\boldsymbol{x}), \boldsymbol{u}_{j,k}, \mathit{maxiter} = 2, \boldsymbol{u}^u_{j,k} \in \overline{\mathcal{D}}),\\ \boldsymbol{u}^b_{j,k} &:= \text{L-BFGS-B}(f(\boldsymbol{x}), \boldsymbol{u}_{j,k}, \mathit{maxiter} = 2, \boldsymbol{u}^b_{j,k} \in \text{"trials cuboid"}),\end{aligned}$$

where **L-BFGS-B** is the modification of Broyden-Fletcher-Goldfarb-Shanno algorithm for local optimization with **l**imited-memory use and **b**ounds on the variables.

We have performed tests and comparison of these modifications and the unmodified DE method over the following functions:

Name	Definition of the test function	Test in domain
Walther type	$f(\boldsymbol{x}) = \exp\left(\exp\left(\prod_{i=1}^{d} x_i^2\right)\right)$,	$\boldsymbol{x} \in [-100, 100]^d$;
Michalewicz	$f(\boldsymbol{x}) = -\sum_{i=1}^{d} \sin(x_i)\sin^{20}\left(\frac{i\,x_i^2}{\pi}\right)$,	$\boldsymbol{x} \in [-2, 2]^d$;
Ackley	$f(\boldsymbol{x}) = -10\exp\left(-0.2\sqrt{\frac{1}{d}\sum_{i=1}^{d} x_i^2}\right)$	
	$-\exp\left(\frac{1}{d}\sum_{i=1}^{d}\cos(2x_i)\right) + 10 + \exp(1)$,	$\boldsymbol{x} \in [-100, 100]^d$;
Periodic	$f(\boldsymbol{x}) = 1 + \sum_{i=1}^{d}\sin^2(x_i) - 0.1\exp\left(-\sum_{i=1}^{d} x_i^2\right)$,	$\boldsymbol{x} \in [-10, 10]^d$.

Table 1. Test functions.

The Walther type function comes from our experience with similar functions in [13], while the other 3 functions are part of standardly used functions in studies of global optimization [9]. The parameters of the test are:

- Dimension – $d = 4$;
- Initial population –$NP = 20$ (very small initial population!);
- Values of the remaining DE parameters:
  - $F = 0.9,\ CR = 0.2$, rand1bin[1];
  - $F = 0.9,\ CR = 0.4$, rand1bin;
  - $F = 0.9,\ CR = 0.4$, best1bin[2];
- Maximum allowed iterations – 300;
- 30 repetitions per experiment.

Summary of the tests is given in Table 2. It is evident that in the examined cases the proposed modifications give better results in terms of mean, but it takes longer time to get result (time is given in milliseconds). Detailed calculations are provided in the Appendix.

	DE								
	rand1bin CR = 0.2, F = 0.9			rand1bin CR = 0.4, F = 0.9			best1bin CR = 0.5, F = 0.9		
	Population		Time	Population		Time	Population		Time
	Mean	St.D.	Mean	Mean	St.D.	Mean	Mean	St.D.	Mean
Walther	inf		02120	inf		021203	inf		01370
Michalewicz	1.4E-3	1.6E-3	00284	4.3E-3	2.6E-3	00278	1.5E-2	1.4E-2	00163
Ackley	2.8E+0	1.1E+0	01208	3.6E+0	1.5E+0	01209	5.1E-5	2.0E-4	01225
Periodic	6.7E-2	4.3E-2	01079	6.9E-2	3.5E-2	01288	9.3E-2	2.5E-2	00625
	L-BFGS-B refinements in $\overline{\mathcal{D}}$								
	rand1bin CR = 0.2, F = 0.9			rand1bin CR = 0.4, F = 0.9			best1bin CR = 0.5, F = 0.9		
	Population		Time	Population		Time	Population		Time
	Mean	St.D.	Mean	Mean	St.D.	Mean	Mean	St.D.	Mean
Walther	**1.1E-14**		00127	**2.96E-17**		00117	**0.0E+0**		00115
Michalewicz	**1.3E-3**	9.0E-4	03275	3.6E-3	2.9E-3	03902	8.8E-3	1.4E-2	02673
Ackley	**0.0E+0**	0.0RE+0	04044	**0.0E+0**	0.0E+0	01960	**0.0E+0**	0.0E+0	01583
Periodic	**4.4E-7**	6.9E-7	00139	**1.4E-7**	1.7E-7	00173	**2.3E-8**	2.5E-8	00167
	L-BFGS-B refinements in trials cuboid								
	rand1bin CR = 0.2, F = 0.9			rand1bin CR = 0.4, F = 0.9			best1bin CR = 0.5, F = 0.9		
	Population		Time	Population		Time	Population		Time
	Mean	St.D.	Mean	Mean	St.D.	Mean	Mean	St.D.	Mean
Walther	7.5E-10		00435	6.8E-11		00339	3.9E-10		00334
Michalewicz	1.8E-3	1.1E-3	03523	**3.5E-3**	3.2E-3	03438	**8.7E-3**	8.9E-3	01917
Ackley	1.0E-2	3.5E-3	20767	1.2E-3	4.9E-4	18709	4.8E-9	4.6E-9	15806
Periodic	1.1E-4	8.0E-5	18120	7.2E-4	4.1E-4	07864	1.0E-2	3.0E-2	04024

Table 2. Summary of the calculations for $d = 4$.

[1]in rand1bin strategy vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ are chosen randomly, as described in Section 2.

[2]in best1bin strategy the vector $\boldsymbol{a}$ is the agent at which the objective function has minimal value. This is widely used and accepted modification.

4. Conclusions

This work proposes a strategy for refinements of the crossover selections in the DE method and compares it with the classical differential evolution algorithm (see [12]). The proposed refinements have both mathematical simplicity (they add an use of a well-known gradient method) and flexibility for exploring broader adjustments (the standard adjustments of DEM and the adjustments for the introduced gradient method). The adjustment of control parameters is a global behavior (and to our knowledge at present there is no a general theory to control the DE parameters in the evolution process). It has been shown that in many cases the proposed strategy is preferable. Some disadvantages are that it involves more control parameters and (of course) high processor time for algorithm to complete.

5. Appendix

Walther	DE					
d=4	Rand1bin, F=0.9, CR=0.2, NP=20		Rand1bin, F=0.9, CR=0.4, NP=20		best1bin, F=0.9, CR=0.4, NP=20	
		time		time		time
	inf	2610.697	inf	2610.697	8.41E-03	995.0240
	inf	2383.008	inf	2383.008	8.49E-01	1393.365
	25.9E-04	1614.215	2.59E-03	1614.215	3.16E-03	1370.036
	inf	2382.627	inf	2382.627	1.79E-03	1196.839
	inf	2394.103	inf	2394.103	inf	1556.650
	08.2E-02	2170.617	8.25E-02	2170.617	5.02E-03	1365.859
	23.8E-04	1866.189	2.38E-03	1866.189	inf	1563.824
	inf	2379.601	inf	2379.601	inf	1554.590
	04.9E-08	1569.562	4.90E-08	1569.562	inf	1561.128
	inf	2399.436	inf	2399.436	1.03E-01	1175.504
	inf	2400.464	inf	2400.464	inf	1551.853
	01.0E-04	1550.847	1.02E-04	1550.847	inf	1552.249
	inf	2380.648	inf	2380.648	inf	1557.912
	42.4E-02	2248.445	4.24E-01	2248.445	inf	1550.933
	04.4E-04	1969.676	4.44E-04	1969.676	1.07E-01	995.2400
	48.9E-04	1462.906	4.89E-03	1462.906	inf	1554.148
	inf	2377.683	inf	2377.683	2.72E-03	1004.234
	42.0E-06	1461.013	4.20E-05	1461.013	inf	1553.815
	03.0E-04	1859.090	3.05E-04	1859.090	inf	1562.357
	inf	2379.121	inf	2379.121	6.71E-02	992.7380
	03.7E-04	1605.049	3.70E-04	1605.049	1.91E-02	1007.976
	01.4E-02	1718.405	1.41E-02	1718.405	inf	1548.751
	inf	2394.772	inf	2394.772	1.23E-06	1286.003
	inf	2379.762	inf	2379.762	1.99E-05	1108.718
	inf	2374.690	inf	2374.690	inf	1552.797
	inf	2388.070	inf	2388.070	inf	1549.196
	01.8E+00	1908.003	1.79E+00	1908.003	inf	1550.097
	inf	2595.840	inf	2595.840	inf	1551.514
	01.4E-06	1983.104	1.37E-06	1983.104	7.12E+10	1142.869
	inf	2504.585	inf	2504.585	3.39E-05	1300.114
mean	**inf**	**2.12E+03**	**inf**	**2.12E+03**	**inf**	**1.37E+03**
Walther	**L-BFGS-B refinements in $\overline{\mathcal{D}}$**					
d=4	Rand1bin, F=0.9, CR=0.2, NP=20		Rand1bin, F=0.9, CR=0.4, NP=20		best1bin, F=0.9, CR=0.4, NP=20	
		time		time		time
	0.00E+00	303.551	0.00E+00	71.920	0.00E+00	122.441
	3.20E-13	79.484	0.00E+00	72.400	0.00E+00	176.021
	0.00E+00	131.27	0.00E+00	126.070	0.00E+00	125.148
	0.00E+00	179.067	0.00E+00	74.0120	0.00E+00	74.6740
	0.00E+00	130.52	0.00E+00	124.484	0.00E+00	122.501
	0.00E+00	129.111	0.00E+00	125.114	0.00E+00	121.675
	0.00E+00	125.957	0.00E+00	127.672	0.00E+00	176.617
	0.00E+00	75.726	0.00E+00	123.866	0.00E+00	123.201
	0.00E+00	128.622	0.00E+00	175.576	0.00E+00	75.117
	0.00E+00	125.377	0.00E+00	123.758	0.00E+00	124.074
	0.00E+00	130.060	0.00E+00	124.863	0.00E+00	124.632
	0.00E+00	175.815	0.00E+00	72.260	0.00E+00	121.760
	0.00E+00	127.794	0.00E+00	125.812	0.00E+00	71.112
	0.00E+00	130.031	0.00E+00	130.597	0.00E+00	125.383
	0.00E+00	76.912	0.00E+00	129.905	0.00E+00	72.394
	0.00E+00	126.594	0.00E+00	75.217	0.00E+00	123.971
	1.78E-15	78.075	0.00E+00	74.441	0.00E+00	125.761
	0.00E+00	77.322	0.00E+00	128.752	0.00E+00	179.299

	0.00E+00	77.345	0.00E+00	125.444	0.00E+00	122.937
	4.44E-15	75.840	0.00E+00	125.728	0.00E+00	120.722
	8.88E-16	75.660	0.00E+00	124.914	0.00E+00	122.213
	0.00E+00	125.991	0.00E+00	176.736	0.00E+00	122.622
	0.00E+00	130.861	0.00E+00	129.896	0.00E+00	72.836
	1.33E-15	74.583	8.88E-16	74.731	0.00E+00	71.089
	0.00E+00	127.639	0.00E+00	125.694	0.00E+00	121.673
	0.00E+00	179.480	0.00E+00	175.531	0.00E+00	73.750
	0.00E+00	181.088	0.00E+00	175.632	0.00E+00	123.982
	0.00E+00	129.315	0.00E+00	124.243	0.00E+00	122.840
	0.00E+00	128.004	0.00E+00	76.333	0.00E+00	121.845
	0.00E+00	177.306	0.00E+00	72.877	0.00E+00	77.423
mean	**1.10E-14**	**1.27E+02**	**2.96E-17**	**1.17E+02**	**0.00E+00**	**1.15E+02**
Walther	**L-BFGS-B refinements in the trials cuboid**					
d=4	Rand1bin, F=0.9, CR=0.2, NP=20		Rand1bin, F=0.9, CR=0.4, NP=20		best1bin, F=0.9, CR=0.4, NP=20	
		time		time		time
	1.79E-12	650.259	4.49E-14	412.724	7.99E-15	624.582
	1.40E-11	455.942	8.04E-14	342.781	1.18E-12	326.164
	1.20E-10	308.701	6.98E-11	346.256	3.11E-14	258.604
	6.74E-12	384.648	2.59E-10	348.411	4.26E-11	320.846
	1.39E-12	380.614	8.30E-13	271.872	3.74E-12	323.127
	7.19E-10	443.539	5.32E-12	354.137	9.68E-14	310.365
	5.31E-11	528.792	6.91E-10	342.119	2.00E-14	321.603
	1.15E-10	443.804	3.62E-12	406.459	1.71E-13	376.232
	1.15E-13	526.802	0.00E+00	349.738	4.03E-09	260.157
	2.34E-12	311.717	1.72E-12	348.512	2.74E-11	376.429
	3.04E-13	588.490	5.09E-10	284.322	5.02E-10	309.938
	1.40E-08	310.000	1.61E-10	284.226	1.39E-10	379.174
	1.92E-12	382.070	2.88E-11	343.669	7.18E-11	437.552
	3.18E-09	306.507	6.56E-12	286.813	1.26E-10	322.872
	1.13E-11	374.079	1.32E-12	352.523	4.12E-10	371.737
	2.02E-09	382.981	7.56E-11	343.638	9.57E-10	265.390
	4.04E-12	524.065	1.07E-10	283.900	7.56E-13	262.887
	1.06E-12	522.588	2.29E-12	342.143	6.13E-14	268.417
	5.82E-14	804.538	2.71E-11	533.909	4.26E-09	325.713
	1.25E-09	309.897	9.10E-14	413.581	7.04E-10	318.298
	3.39E-11	303.892	4.11E-11	275.850	3.18E-10	373.819
	3.05E-11	381.215	4.77E-12	354.695	1.30E-13	379.394
	3.43E-10	382.364	5.29E-13	337.529	2.37E-13	323.629
	5.27E-10	379.769	4.22E-14	353.803	5.11E-12	376.163
	9.19E-11	458.439	5.86E-12	340.845	4.47E-11	264.630
	0.00E+00	520.521	2.92E-12	279.384	1.86E-12	252.509
	3.08E-12	384.227	3.15E-11	280.328	1.13E-11	366.052
	3.05E-12	520.230	3.45E-12	281.090	1.68E-11	382.799
	1.33E-15	459.070	1.33E-15	335.712	1.72E-11	268.019
	1.26E-11	306.787	4.69E-13	349.709	2.32E-12	262.446
mean	**7.53E-10**	**4.35E+02**	**6.80E-11**	**3.39E+02**	**3.90E-10**	**3.34E+02**
Periodic	**DE**					
d=4	Rand1bin, F=0.9, CR=0.2, NP=20		Rand1bin, F=0.9, CR=0.4, NP=20		best1bin, F=0.9, CR=0.4, NP=20	
		time		time		time
	10.0E-02	1140.167	1.01E-01	1290.823	1.01E-01	580.472
	05.3E-02	1011.619	1.01E-01	1289.174	1.00E-01	531.535
	14.6E-04	1277.207	8.81E-02	1294.317	1.01E-01	708.863
	10.0E-02	901.165	4.19E-03	1286.949	1.01E-01	537.901
	10.0E-02	953.821	8.76E-02	1295.461	1.00E-01	686.109
	10.1E-02	886.330	1.01E-01	1292.304	1.00E-01	588.695
	10.0E-02	1035.971	1.00E-01	1293.229	1.01E-01	568.116
	10.0E-02	1028.265	1.74E-02	1288.282	1.01E-01	654.594
	10.0E-02	933.613	6.28E-02	1285.146	1.00E-01	487.999
	10.9E-04	1280.021	1.66E-02	1284.823	1.01E-01	765.816
	01.8E-02	1279.926	1.00E-01	1288.447	1.00E-01	857.501
	10.0E-02	1120.272	1.01E-01	1288.236	1.00E-01	525.264
	10.1E-02	913.839	2.26E-02	1286.218	1.00E-01	638.937
	02.3E-02	1277.683	1.02E-01	1295.954	1.00E-01	590.840
	10.0E-02	1061.736	1.00E-01	1291.984	1.86E-04	705.933
	07.9E-02	919.798	1.01E-01	1292.540	1.00E-01	664.829
	10.0E-02	982.507	1.70E-02	1285.213	1.01E-01	673.815
	01.7E-02	1279.734	1.01E-01	1262.492	1.01E-01	618.322
	10.0E-02	976.809	4.15E-02	1283.622	1.01E-01	551.694
	65.0E-04	1286.168	1.00E-01	1287.397	1.00E-01	660.955
	09.3E-02	835.985	7.12E-02	1278.664	1.57E-04	636.287
	10.0E-02	837.648	1.00E-01	1292.014	1.00E-01	603.874
	35.3E-04	1282.227	2.32E-02	1302.326	1.01E-01	554.877
	70.4E-04	1275.881	7.34E-02	1288.919	1.01E-01	678.603
	10.1E-02	1087.733	6.56E-02	1285.314	1.00E-01	561.656
	10.0E-02	988.464	1.42E-02	1287.566	1.01E-01	649.331
	07.1E-04	1290.162	3.55E-02	1290.737	1.01E-01	657.634
	10.0E-02	1060.513	4.97E-02	1288.935	1.00E-01	618.037
	01.1E-02	1274.430	7.54E-02	1285.825	1.01E-01	667.797
	10.1E-02	896.917	1.01E-01	1283.552	1.01E-01	550.820
mean	**6.74E-02**	**1.08E+03**	**6.92E-02**	**1.29E+03**	**9.39E-02**	**6.26E+02**
Periodic	**L-BFGS-B refinements in $\overline{\mathcal{D}}$**					
d=4	Rand1bin, F=0.9, CR=0.2, NP=20		Rand1bin, F=0.9, CR=0.4, NP=20		best1bin, F=0.9, CR=0.4, NP=20	
		time		time		time
	9.15E-08	381.026	4.47E-07	158.248	7.56E-09	157.867
	1.05E-06	160.178	5.49E-08	157.771	7.09E-08	157.832
	6.65E-08	160.601	1.71E-08	157.965	4.29E-08	157.846
	5.78E-07	159.003	4.09E-07	157.218	2.74E-08	158.437

	2.94E-10	237.343	2.19E-07	157.007	4.83E-08	157.536
	6.19E-10	235.248	4.51E-10	235.081	2.83E-11	235.294
	1.72E-06	157.337	5.54E-11	234.831	3.45E-09	158.037
	1.72E-07	157.516	3.57E-07	158.255	2.21E-11	234.070
	2.36E-06	157.914	2.94E-09	159.075	2.96E-08	158.430
	8.42E-11	234.584	5.31E-08	158.998	1.43E-12	235.010
	1.24E-09	235.927	2.16E-10	237.037	3.39E-08	157.684
	6.41E-11	233.932	5.45E-08	158.622	1.46E-08	158.075
	4.94E-07	157.297	4.30E-08	159.875	1.19E-08	156.961
	4.01E-10	233.664	5.26E-07	159.279	4.10E-08	157.479
	5.56E-10	234.909	3.86E-07	158.654	3.59E-08	157.509
	1.79E-07	157.491	1.29E-07	159.232	1.20E-08	156.868
	4.43E-07	157.912	7.48E-11	236.515	1.28E-08	156.743
	7.58E-07	156.995	1.25E-07	158.502	5.10E-11	234.346
	1.59E-07	158.522	2.73E-12	235.890	1.22E-08	156.840
	2.94E-07	156.713	2.05E-08	158.870	1.39E-08	157.077
	5.33E-07	157.560	3.98E-11	235.437	1.34E-08	156.934
	1.30E-07	156.082	1.79E-07	158.240	3.12E-08	157.443
	1.27E-07	158.166	3.90E-07	158.507	1.86E-08	158.436
	9.90E-11	235.035	1.81E-07	158.758	2.44E-08	157.135
	1.54E-07	157.702	3.61E-07	159.335	2.85E-08	157.455
	2.06E-06	158.217	1.60E-10	236.091	1.37E-08	157.799
	1.78E-07	157.265	2.42E-07	159.164	5.43E-09	157.954
	2.21E-06	156.700	1.22E-08	157.821	1.22E-07	157.753
	1.84E-07	158.299	4.06E-07	158.366	3.21E-09	157.637
	7.24E-08	157.449	4.43E-08	158.179	1.67E-08	158.345
mean	**4.67E-07**	**1.86E+02**	**1.55E-07**	**1.77E+02**	**2.32E-08**	**1.68E+02**
Periodic	**L-BFGS-B refinements in the trials cuboid**					
d=4	Rand1bin, F=0.9, CR=0.2, NP=20		Rand1bin, F=0.9, CR=0.4, NP=20		best1bin, F=0.9, CR=0.4, NP=20	
		time		time		time
	1.46E-04	18896.880	4.91E-04	7102.206	4.74E-04	3973.559
	1.04E-04	18614.042	8.46E-04	7105.758	6.28E-04	4197.469
	8.44E-05	17353.499	7.00E-04	6797.998	3.79E-04	3422.193
	3.29E-05	18431.747	5.25E-04	7814.264	6.48E-04	4047.478
	6.37E-05	18651.132	4.81E-04	7848.582	4.90E-04	4771.754
	1.11E-04	18551.722	5.99E-04	6632.892	5.81E-04	2984.932
	2.71E-04	17302.092	4.17E-04	9429.805	7.08E-04	3370.723
	1.34E-04	13811.509	1.70E-04	6550.365	3.10E-04	3309.105
	2.75E-04	18739.162	9.28E-04	6563.725	1.13E-03	3377.632
	9.23E-05	18740.209	1.42E-03	8174.174	3.57E-04	4609.065
	1.61E-04	17659.838	1.69E-03	7151.703	7.20E-04	3552.428
	7.47E-05	18462.965	9.57E-04	8181.484	2.33E-04	3656.714
	8.27E-05	18556.382	6.24E-04	9343.822	2.35E-04	4405.634
	3.51E-05	18693.155	5.64E-04	7182.324	1.46E-03	3460.708
	3.39E-04	14332.073	3.04E-04	10703.468	9.71E-04	3878.857
	8.03E-05	18532.010	6.94E-04	8481.807	8.56E-04	3865.665
	4.35E-05	18546.305	1.57E-03	7849.014	3.03E-04	3942.891
	1.74E-04	18662.928	9.38E-04	9527.965	1.49E-05	4919.926
	6.14E-05	18557.396	2.57E-04	7325.417	1.53E-03	3187.234
	1.25E-04	17384.699	8.32E-04	7513.753	1.00E-01	6108.161
	2.56E-04	18606.211	1.31E-04	9348.994	7.29E-04	4169.817
	1.21E-05	17969.410	3.99E-04	7029.860	8.73E-04	3407.222
	1.86E-04	18639.811	4.72E-04	7305.188	2.48E-04	3263.714
	5.16E-05	18712.021	1.26E-03	7887.428	6.96E-04	5648.769
	7.38E-05	18572.774	6.56E-04	8695.142	1.00E-01	6040.359
	4.78E-05	18533.669	3.19E-04	7769.133	1.05E-03	3306.108
	1.01E-04	18537.131	1.10E-03	7869.69	7.01E-04	4092.971
	9.53E-05	18388.743	1.11E-03	7507.688	1.00E-01	4062.154
	1.06E-04	18648.415	1.00E-03	8017.874	4.54E-04	4432.809
	5.81E-05	18533.216	1.89E-04	7215.147	3.01E-04	3264.573
mean	**1.16E-04**	**1.81E+04**	**7.22E-04**	**7.86E+03**	**1.06E-02**	**4.02E+03**
d=4	Rand1bin, F=0.9, CR=0.2, NP=20		Rand1bin, F=0.9, CR=0.4, NP=20		best1bin, F=0.9, CR=0.4, NP=20	
		time		time		time
	03.4E+00	1229.535	4.86E+00	1228.657	1.01E-06	1226.825
	03.7E+00	1064.625	5.04E+00	1220.671	4.91E-06	1227.322
	03.5E+00	1061.339	5.05E+00	1223.483	4.21E-07	1229.334
	02.8E+00	1058.486	1.72E+00	1238.028	2.68E-06	1241.192
	01.4E+00	1057.079	3.88E+00	1234.379	3.01E-06	1224.690
	02.1E+00	1057.802	5.77E+00	1232.957	1.97E-06	1229.414
	04.6E+00	1055.766	2.73E+00	1223.461	1.18E-07	1240.158
	02.7E+00	1059.603	4.41E+00	1232.158	3.82E-05	1226.006
	04.2E+00	1054.752	5.15E+00	1233.587	1.93E-07	1250.389
	89.1E-02	1056.091	4.52E+00	1236.523	1.19E-07	1238.552
	03.7E+00	1055.218	2.29E+00	1226.052	1.47E-06	1229.686
	01.0E+00	1213.675	1.27E+00	1226.164	4.55E-07	1237.154
	02.8E+00	1111.810	4.68E+00	1231.079	1.16E-05	1236.100
	04.4E+00	1109.427	4.19E+00	1225.495	2.02E-02	1225.465
	02.1E+00	1118.035	3.58E+00	1230.049	2.79E-04	1227.692
	04.1E+00	1118.242	5.35E+00	1232.776	1.75E-05	1232.121
	03.3E+00	1110.173	4.61E+00	1225.351	3.06E-06	1238.126
	03.7E+00	1115.914	4.24E+00	1226.921	1.87E-06	1235.717
	03.3E+00	1357.503	4.77E+00	1227.075	9.61E-04	1234.578
	03.2E+00	1231.343	4.09E+00	1232.028	8.56E-07	1229.399
	03.6E+00	1225.145	3.36E+00	1228.953	2.25E-07	1230.460
	02.1E+00	1222.422	4.45E+00	1236.548	5.54E-05	1222.137
	02.7E+00	1222.589	3.80E+00	1239.267	7.17E-02	1225.722
	02.8E+00	1221.872	1.14E+00	1227.951	1.96E-08	1227.677
	02.7E+00	1224.396	4.60E+00	1242.141	7.17E-04	1236.660
	04.8E+00	1233.585	3.18E+00	1233.560	5.26E-05	1227.937

	02.8E+00	1225.668	4.29E+00	1242.303	9.55E-07	1229.030
	02.3E+00	1231.106	9.28E-01	1219.872	2.61E-05	1226.621
	01.3E+00	1210.602	2.59E+00	1233.039	1.21E-04	1223.077
	03.8E+00	1217.455	4.25E+00	1226.352	1.36E-06	1233.772
mean	**2.99E+00**	**1.15E+03**	**3.83E+00**	**1.23E+03**	**3.14E-03**	**1.23E+03**
d=4	Rand1bin, F=0.9, CR=0.2, NP=20	time	Rand1bin, F=0.9, CR=0.4, NP=20	time	best1bin, F=0.9, CR=0.4, NP=20	time
	4.44E-16	3786.76	4.44E-16	2046.184	4.44E-16	1684.774
	4.44E-16	7650.850	4.44E-16	1527.091	4.44E-16	1466.397
	4.44E-16	3692.850	4.44E-16	1991.732	4.44E-16	2593.692
	4.44E-16	4680.865	4.44E-16	1692.099	4.44E-16	1246.259
	4.44E-16	2194.641	4.44E-16	2127.670	4.44E-16	1530.420
	4.44E-16	2660.962	4.44E-16	1751.040	4.44E-16	1177.945
	4.44E-16	3121.989	4.44E-16	2067.634	4.44E-16	1916.112
	4.44E-16	4370.847	4.44E-16	1696.497	4.44E-16	1911.013
	4.44E-16	1994.737	4.44E-16	1602.422	4.44E-16	1335.135
	4.44E-16	3471.404	4.44E-16	1611.413	4.44E-16	1550.081
	4.44E-16	5568.382	4.44E-16	1680.642	4.44E-16	1766.108
	4.44E-16	2651.570	4.44E-16	2438.616	4.44E-16	1108.128
	4.44E-16	2583.710	4.44E-16	1689.749	4.44E-16	1740.288
	4.44E-16	2354.667	4.44E-16	1316.854	4.44E-16	1241.922
	4.44E-16	7009.884	4.44E-16	2208.603	4.44E-16	1612.894
	4.44E-16	2713.276	4.44E-16	2305.846	4.44E-16	1188.272
	4.44E-16	2717.536	4.44E-16	3947.035	4.44E-16	1709.014
	4.44E-16	8668.264	4.44E-16	1479.180	4.44E-16	1240.187
	4.44E-16	2742.589	4.44E-16	1625.292	4.44E-16	1169.085
	4.44E-16	3407.442	4.44E-16	2145.363	4.44E-16	1844.525
	4.44E-16	2206.415	4.44E-16	1900.349	4.44E-16	2307.075
	4.44E-16	14932.293	4.44E-16	1896.732	4.44E-16	1475.917
	4.44E-16	2082.505	4.44E-16	2590.215	4.44E-16	1548.632
	4.44E-16	1766.250	4.44E-16	1608.510	4.44E-16	1418.614
	4.44E-16	3635.780	4.44E-16	1844.274	4.44E-16	1498.387
	4.44E-16	4383.185	4.44E-16	2286.510	4.44E-16	2141.821
	4.44E-16	3772.552	4.44E-16	1698.840	4.44E-16	1182.165
	4.44E-16	2712.093	4.44E-16	2063.455	4.44E-16	1740.819
	4.44E-16	6002.199	4.44E-16	1989.879	4.44E-16	1332.668
	4.44E-16	1812.604	4.44E-16	1973.940	4.44E-16	1813.705
mean	**4.44E-16**	**4.04E+03**	**4.44E-16**	**1.96E+03**	**4.44E-16**	**1.58E+03**
Ackley	**L-BFGS-B refinements in the trials cuboid**					
d=4	Rand1bin, F=0.9, CR=0.2, NP=20	time	Rand1bin, F=0.9, CR=0.4, NP=20	time	best1bin, F=0.9, CR=0.4, NP=20	time
	1.46E-04	18896.880	4.91E-04	7102.206	4.74E-04	3973.559
	1.04E-04	18614.042	8.46E-04	7105.758	6.28E-04	4197.469
	8.44E-05	17353.499	7.00E-04	6797.998	3.79E-04	3422.193
	3.29E-05	18431.747	5.25E-04	7814.264	6.48E-04	4047.478
	6.37E-05	18651.132	4.81E-04	7848.582	4.90E-04	4771.754
	1.11E-04	18551.722	5.99E-04	6632.892	5.81E-04	2984.932
	2.71E-04	17302.092	4.17E-04	9429.805	7.08E-04	3370.723
	1.34E-04	13811.509	1.70E-04	6550.365	3.10E-04	3309.105
	2.75E-04	18739.162	9.28E-04	6563.725	1.13E-03	3377.632
	9.23E-05	18740.209	1.42E-03	8174.174	3.57E-04	4609.065
	1.61E-04	17659.838	1.69E-03	7151.703	7.20E-04	3552.428
	7.47E-05	18462.965	9.57E-04	8181.484	2.33E-04	3656.714
	8.27E-05	18556.382	6.24E-04	9343.822	2.35E-04	4405.634
	3.51E-05	18693.155	5.64E-04	7182.324	1.46E-03	3460.708
	3.39E-04	14332.073	3.04E-04	10703.468	9.71E-04	3878.857
	8.03E-05	18532.010	6.94E-04	8481.807	8.56E-04	3865.665
	4.35E-05	18546.305	1.57E-03	7849.014	3.03E-04	3942.891
	1.74E-04	18662.928	9.38E-04	9527.965	1.49E-05	4919.926
	6.14E-05	18557.396	2.57E-04	7325.417	1.53E-03	3187.234
	1.25E-04	17384.699	8.32E-04	7513.753	1.00E-01	6108.161
	2.56E-04	18606.211	1.31E-04	9348.994	7.29E-04	4169.817
	1.21E-05	17969.410	3.99E-04	7029.860	8.73E-04	3407.222
	1.86E-04	18639.811	4.72E-04	7305.188	2.48E-04	3263.714
	5.16E-05	18712.021	1.26E-03	7887.428	6.96E-04	5648.769
	7.38E-05	18572.774	6.56E-04	8695.142	1.00E-01	6040.359
	4.78E-05	18533.669	3.19E-04	7769.133	1.05E-03	3306.108
	1.01E-04	18537.131	1.10E-03	7869.690	7.01E-04	4092.971
	9.53E-05	18388.743	1.11E-03	7507.688	1.00E-01	4062.154
	1.06E-04	18648.415	1.00E-03	8017.874	4.54E-04	4432.809
	5.81E-05	18533.216	1.89E-04	7215.147	3.01E-04	3264.573
mean	**1.16E-04**	**1.81E+04**	**7.22E-04**	**7.86E+03**	**1.06E-02**	**4.02E+03**
Michalewicz	**DE**					
d=4	Rand1bin, F=0.9, CR=0.2, NP=20	time	Rand1bin, F=0.9, CR=0.4, NP=20	time	best1bin, F=0.9, CR=0.4, NP=20	time
	02.4E-04	439.729	3.61E-03	264.771	4.51E-03	171.639
	10.7E-04	310.222	2.14E-03	308.687	1.70E-03	180.596
	05.4E-04	346.621	3.88E-03	295.420	3.28E-03	178.740
	06.3E-04	290.628	9.93E-03	251.501	3.83E-03	190.052
	12.8E-04	286.695	5.14E-03	273.542	3.38E-03	171.464
	46.9E-04	252.532	3.18E-03	299.326	3.31E-04	175.602
	11.2E-04	286.056	2.45E-03	284.963	4.01E-03	153.019
	02.8E-04	273.195	3.09E-03	315.107	2.07E-03	153.384
	22.8E-04	266.291	4.87E-03	286.285	3.10E-03	184.116
	01.1E-04	278.361	3.86E-03	288.237	2.86E-03	197.638
	04.4E-04	310.341	1.72E-03	287.607	2.63E-03	166.022
	05.8E-04	263.431	1.57E-03	305.615	1.18E-03	170.742
	24.8E-04	280.034	1.25E-03	248.675	2.44E-03	167.429

	15.1E-04	274.465	7.67E-03	275.814	4.37E-02	167.684
	29.8E-04	296.353	5.30E-03	271.419	4.35E-02	161.762
	05.7E-04	278.402	7.06E-03	270.198	1.01E-04	188.666
	02.7E-04	289.362	1.91E-03	265.359	2.86E-03	174.950
	05.9E-04	263.388	5.96E-03	254.779	4.60E-02	145.589
	13.2E-04	283.463	2.75E-03	294.220	2.00E-03	153.439
	14.1E-04	247.552	3.26E-03	268.074	6.78E-03	158.934
	14.3E-04	315.189	5.69E-03	247.654	4.50E-03	156.711
	06.0E-04	275.504	9.55E-03	280.958	4.44E-02	148.068
	15.1E-04	274.334	4.34E-05	291.688	4.29E-03	170.508
	06.8E-04	264.545	3.80E-03	270.061	2.52E-03	166.583
	23.5E-04	246.840	1.97E-03	275.650	2.60E-04	159.711
	05.5E-04	283.266	1.07E-02	270.970	3.05E-03	162.074
	07.1E-04	261.440	4.98E-03	284.337	3.89E-03	179.247
	80.5E-04	236.614	1.92E-03	271.354	5.84E-03	187.998
	03.6E-04	287.335	3.38E-03	270.086	2.34E-03	161.981
	22.9E-04	265.053	1.65E-03	279.108	1.25E-03	166.294
mean	**1.43E-03**	**2.84E+02**	**4.14E-03**	**2.78E+02**	**8.42E-03**	**1.69E+02**
Michalewicz	**L-BFGS-B refinements in $\overline{\mathcal{D}}$**					
d=4	Rand1bin, F=0.9, CR=0.2, NP=20		Rand1bin, F=0.9, CR=0.4, NP=20		best1bin, F=0.9, CR=0.4, NP=20	
		time		time		time
	3.67E-03	3051.853	5.08E-03	3810.535	4.32E-02	1746.578
	1.86E-03	2843.193	3.23E-03	3308.467	2.64E-03	2760.472
	2.55E-03	2782.262	1.38E-03	3688.374	1.67E-03	3064.264
	3.41E-03	2818.485	8.85E-03	4078.401	4.32E-02	1619.376
	5.65E-04	3291.540	7.06E-04	3726.590	4.50E-03	3513.011
	2.57E-03	3599.211	1.46E-03	4063.710	2.04E-03	2681.574
	2.46E-04	3211.052	4.08E-03	4117.542	2.12E-03	3271.224
	1.25E-03	3120.922	1.69E-03	3555.606	2.71E-03	2828.479
	3.46E-04	3469.162	4.44E-03	4355.244	5.02E-03	2656.644
	1.39E-03	3157.277	3.27E-03	4022.664	2.86E-03	2527.007
	4.19E-04	3313.627	7.09E-03	3313.672	4.95E-02	2095.322
	6.72E-04	3339.517	2.10E-03	3878.963	1.58E-03	2708.593
	5.28E-04	3552.425	4.46E-03	4227.241	8.90E-03	2387.935
	4.33E-04	3531.444	1.16E-03	3856.955	4.47E-02	2028.009
	1.07E-03	3806.400	3.70E-03	3982.247	2.11E-03	2086.313
	7.92E-04	3457.661	6.99E-03	4143.160	5.65E-03	2603.249
	1.19E-03	3348.003	2.10E-03	3964.865	2.08E-03	3469.617
	7.76E-04	3225.685	2.66E-03	4056.368	3.64E-03	2687.701
	1.52E-03	3377.139	3.73E-03	3935.404	6.63E-03	2740.814
	8.84E-04	3367.460	1.24E-03	4044.062	1.96E-03	2693.660
	1.20E-03	3534.464	2.71E-03	3702.116	2.37E-03	2651.414
	2.49E-03	3302.463	4.89E-03	3862.620	1.49E-03	2803.161
	1.49E-03	3472.087	6.41E-04	3897.318	4.29E-03	3012.670
	2.13E-03	3198.258	1.23E-02	3824.705	1.21E-03	2296.670
	1.46E-03	3419.288	2.82E-03	3880.059	2.05E-03	3332.942
	1.63E-03	3346.244	3.38E-04	4179.230	7.19E-04	2910.289
	5.35E-04	3223.584	1.02E-02	3797.558	5.56E-03	2574.071
	9.97E-04	2856.323	2.63E-03	3590.365	1.43E-03	3200.111
	1.42E-03	3035.563	2.41E-03	4306.688	3.03E-03	2674.495
	2.53E-04	3198.027	5.90E-04	3894.337	6.64E-03	2584.904
mean	**1.33E-03**	**3.28E+03**	**3.63E-03**	**3.90E+03**	**8.85E-03**	**2.67E+03**
Michalewicz	**L-BFGS-B refinements in the trials cuboid**					
d=4	Rand1bin, F=0.9, CR=0.2, NP=20		Rand1bin, F=0.9, CR=0.4, NP=20		best1bin, F=0.9, CR=0.4, NP=20	
		time		time		time
	2.14E-03	3586.427	4.97E-04	3894.564	1.45E-03	2227.526
	5.88E-04	3520.054	2.31E-03	3544.678	3.41E-03	1917.075
	2.70E-03	3373.392	4.27E-03	3401.154	1.55E-03	1911.879
	2.20E-03	3722.356	6.41E-03	3432.405	1.70E-03	2021.587
	2.95E-03	3520.751	4.81E-04	3500.625	2.40E-03	1833.522
	1.60E-03	3581.145	8.88E-03	3430.892	3.46E-03	1909.649
	2.21E-03	3590.646	1.74E-02	3118.104	8.78E-04	2024.389
	3.60E-03	3667.731	3.11E-03	3722.941	5.37E-03	1775.601
	2.51E-03	3191.967	1.43E-03	3532.816	1.73E-03	1902.920
	1.66E-03	3168.823	5.14E-03	3137.244	7.07E-03	1928.161
	7.45E-04	3719.485	6.88E-03	3336.263	1.14E-02	2003.169
	9.44E-04	3331.165	4.14E-03	3516.520	4.76E-03	2082.871
	8.07E-04	3401.101	4.11E-03	3748.826	3.52E-03	2049.784
	5.64E-04	4077.901	2.50E-03	3319.199	3.19E-03	1720.819
	4.85E-03	3344.546	4.51E-03	3329.366	9.83E-04	2090.493
	2.45E-03	3690.674	1.10E-03	3411.897	3.99E-03	1929.029
	1.93E-03	3640.174	3.81E-03	3326.764	2.44E-03	2048.018
	2.69E-03	3731.724	4.94E-04	3301.490	2.02E-03	1882.408
	2.85E-04	3390.132	4.58E-03	3031.546	4.70E-02	1651.918
	3.82E-03	3425.509	1.79E-03	3374.928	1.92E-03	1896.015
	1.35E-03	3783.940	2.20E-03	3328.071	4.31E-02	1723.058
	2.04E-03	3704.293	2.40E-03	3189.598	4.49E-03	1951.405
	5.08E-04	3370.600	4.37E-03	3641.346	4.41E-02	1723.876
	1.25E-03	3421.279	1.02E-03	3645.618	2.24E-03	2107.832
	1.90E-03	3525.377	3.06E-03	3534.246	4.42E-02	2014.638
	7.59E-04	3527.844	2.15E-03	3291.791	4.15E-03	1869.944
	8.25E-04	3346.400	4.82E-03	3102.813	4.74E-03	1892.069
	2.68E-03	3219.138	2.38E-03	3611.099	5.64E-03	1918.743
	8.26E-04	3600.028	4.40E-03	3927.808	3.98E-03	1806.578
	3.46E-03	3518.395	2.50E-03	3468.423	2.43E-03	1705.767
mean	**1.89E-03**	**3.52E+03**	**3.77E-03**	**3.44E+03**	**8.98E-03**	**1.92E+03**

Table 3. Benchmark results

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