

NOTE ON THE VOLATILITY STRUCTURE

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Abstract. *The attempts to construct a model that correctly replicates the market realities reached the maturity to challenge the standard Brownian motion (BM) as the stochasticity driver of the Black-Scholes log-returns. Recent research provides arguments to generalize the standard BM with a fractional one (fBM) [2, 3, 4]. The capability of the fBM-based model to simulate long-memory time series explains the quest for the relevant value of the Hurst parameter (H-index). The estimation of the H-index requires the availability of data set for the unobservable volatility. The inputs for the model calibration include high-frequency trades and quotes that are integrated (regularized) to get a reliable proxy for the instantaneous variance. The data span across more than 20 years and cover a large part of the COVID-19 period which allows concluding whether the volatility dependence during turmoil times was of short range or long range. The value of the inferred H-index indicates whether the process was smoother or rougher compared to the Black-Scholes one. These conclusions depend on the data quality [1].*

Key words: volatility structure, fractional Brownian Motion, rough volatility, long memory.

References

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